



NOTI
h
D
Δ

COSA POSSIAMO DIRE:

Pitagora: $x^2 + h^2 = d_1^2$
 $(D-x)^2 + h^2 = d_2^2$

differenza di cammino

$d_1 - d_2 = c \cdot \tau_i = \Delta \Rightarrow d_1 = d_2 + \Delta$
↑ velocità del suono ← ritardo
 $c = 20,1 \sqrt{T} \quad T(K)$

triangoli
rettangoli:

$h = d_1 \sin \alpha$
 $h = d_2 \sin \beta$
 $x = d_1 \cos \alpha$
 $D-x = d_2 \cos \beta$

Carnot: $D^2 = d_1^2 + d_2^2 - 2d_1d_2 \cos(\pi - \alpha - \beta) = d_1^2 + d_2^2 + 2d_1d_2 \cos(\alpha + \beta)$

VOGLIAMO ESPRIMERE X IN FUNZIONE DI h, D, Δ

$x^2 + h^2 = d_2^2 + \Delta^2 + 2\Delta d_2$ (1)

$D^2 + x^2 - 2Dx + h^2 = d_2^2 \rightarrow D^2 - 2Dx + \Delta^2 + 2\Delta d_2 = 0$

$x = \frac{D^2 + \Delta^2 + 2\Delta d_2}{2D}$

Sostituisco x in (1):

$D^4 + \Delta^4 + 4\Delta^2 d_2^2 + 2D^2 \Delta^2 + 4D^2 \Delta d_2 + 4\Delta^3 d_2 + 4D^2 h^2 = 4D^2 d_2^2 + 4D^2 \Delta^2 + 8D^2 \Delta d_2$

$4(\Delta^2 - D^2)d_2^2 + 4(\Delta^3 - D^2 \Delta)d_2 + D^4 + \Delta^4 + 4D^2 h^2 - 2D^2 \Delta^2 = 0$

$d_2 = \frac{-2(\Delta^3 - D^2 \Delta) \pm \sqrt{4(\Delta^3 - D^2 \Delta)^2 - 4(\Delta^2 - D^2)(D^4 + \Delta^4 + 4D^2 h^2 - 2D^2 \Delta^2)}}{4(\Delta^2 - D^2)}$

Nella radice $4\Delta^6 + 4D^4 \Delta^2 - 8\Delta^4 D^2 - 4\Delta^2 D^4 - 4\Delta^6 - 16\Delta^2 D^2 h^2 + 8D^2 \Delta^4$
 $+ 4D^6 + 4D^2 \Delta^4 + 16D^4 h^2 - 8D^4 \Delta^2 =$
 $= 16h^2 D^2 (D^2 - \Delta^2) + 4D^2 (D^4 - 2D^2 \Delta^2 + \Delta^4) =$
 $(D^2 - \Delta^2)^2$

$$= 4D^2(D^2 - \Delta^2) \cdot [4h^2 + D^2 - \Delta^2]$$

$$d_2 = \frac{-2\Delta \cdot (\Delta^2 - D^2) \pm 2D \sqrt{(D^2 - \Delta^2) [4h^2 + D^2 - \Delta^2]}}{4(D^2 - \Delta^2)} =$$

$$= -\frac{\Delta}{2} \pm \frac{D}{2} \sqrt{\frac{4h^2 + D^2 - \Delta^2}{D^2 - \Delta^2}} \quad \text{scelgo } + \text{ (altrimenti } d_2 < 0)$$

Dunque $d_2 = -\frac{\Delta}{2} + \frac{D}{2} \sqrt{\frac{4h^2 + D^2 - \Delta^2}{D^2 - \Delta^2}}$

$$X = \frac{D^2 + \Delta^2 + 2\Delta d_2}{2D}$$